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AN ALGORITHM FOR HIERARCHICAL NETWORK DESIGN

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Abstract—We consider the *network design problem* of locating a set of concentrators which serves a set of customers with known demands. The uncapacitated facility location model is applied to locate the concentrators. Then, for each concentrator we analyze a topological optimization of its subnetwork based on a simple heuristic. In a third phase, we apply the last model to solve the upper level subnetwork connecting the concentrators to a root node. Computational results are reported for three medium-sized cities.

Keywords: Hierarchical network, heuristic, location.

1. INTRODUCTION

The *network design problem* arises in a variety of settings, ranging from telecommunication to transportation planning, which raises issues of dimensioning, topological design and routing (see Boorstyn and Frank, 1977; Tanenbaum, 1981; Gavish, 1982; Magnanti and Wong, 1984; Balakrishnan and Altinkemer, 1992). They have found wide application in computer networks and telecommunication systems.

We consider a centralized concentrator-based network (Gavish, 1992) in a 2-level hierarchical configuration. A set of terminal nodes is served by a set of concentrators which is assigned to the root node. Each arc has a variable and fixed costs and each concentrator has a fixed cost. The objective of the design is to select a subset of concentrators and a subset of arcs that minimizes the sum of variable and fixed costs. We are applying our model in telecommunication network planning, (see Figure 1).

Minoux (1989) and Gavish (1992) give a selective bibliography of network design problems. The model considered here treats the location, dimensioning and topological design aspects in the same problem. It is a cornerstone of many important applications requiring additional constraints, such as channel modularity (Mateus *et al.*, 1990), survivability (Monma and Shallcross, 1989), budget constraint (Dionne and Florian, 1979) and traffic control (Gavish, 1982). Moreover, a host of network models can be viewed as special cases of the problem above, such as the 2-level hierarchical network where all the terminal nodes, concentrators and root node are linked without Steiner nodes. Gavish (1982) describes a formulation for centralized heterogeneous networks with concentrators. In the star-star concentrator location problem (Mirzaian, 1985), all the subnetworks are star networks, and the number of terminals connected to a concentrator must not exceed a positive number. This problem can also be reduced to the capacitated location problem (Tang *et al.*, 1978; Beasley, 1988; Mateus and Bornstein, 1991).

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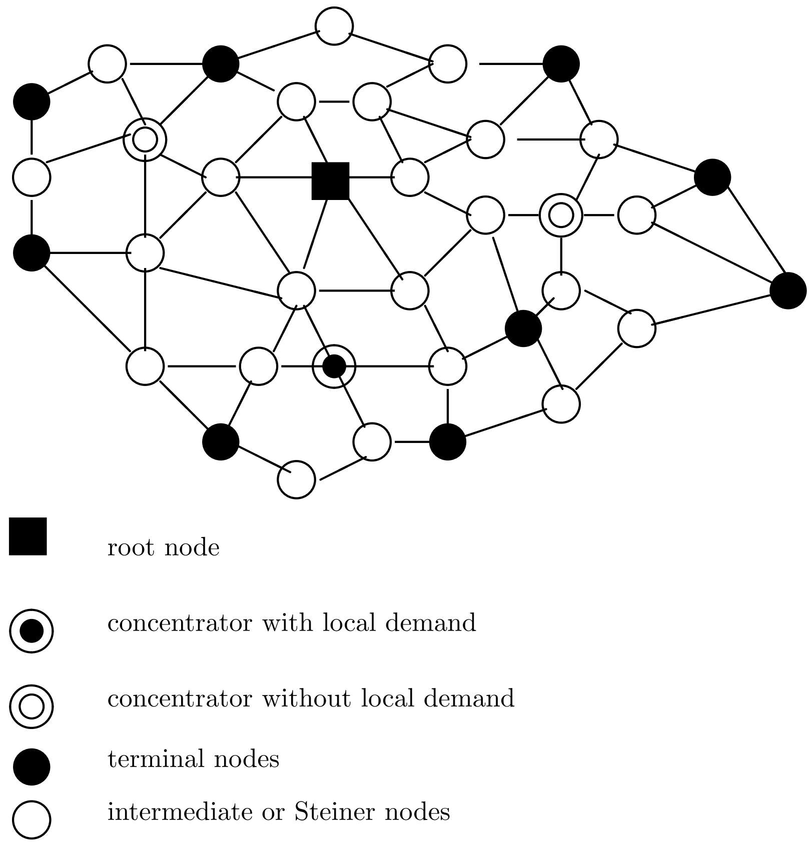

Figure 1. The Hierarchical Network

In both Level-1 and Level-2 subnetworks, a local network is designed (Balakrishnan *et al.*, 1991). Moreover, the local access design can be made according to a Steiner network (Luna *et al.*, 1987; Balakrishnan *et al.*, 1989) or based on the minimal spanning tree problem (Gavish, 1982; Hochbaum and Segev, 1989). The problem studied in this paper explores the Steiner network embedded in the real network. The terminal nodes in Level-2 and the concentrators in Level-1 share the arcs to reduce the connection costs. In fact, the local network problem generalizes the Steiner tree problem on a directed graph. This can be seen if we disregard the variable cost associated with any arc. The problem becomes one of finding a minimum cost tree that contains a directed path between a concentrator p and each terminal node assigned to p, T^p. In this sense, we are solving a NP-hard problem for which some approaches have been devised (Aneja, 1980; Wong, 1984; Maculan, 1987; Beasley, 1989; Jain, 1989). When T^p contains all the nodes of the subnetwork, except p, the problem reduces to the minimal spanning tree (Kruskal, 1956; Prim, 1957).

The local network problem can also be viewed as a generalization of the minimal cost network flow problem (MCNF) with a single source. It becomes a single source MCNF when there is no fixed cost associated with the choice of any arc (Luna *et al.*, 1987). The solution to the special MCNF problem is obtained by finding the shortest paths from p to all terminal nodes in T^p (Dijkstra, 1959).

2. NETWORK DESIGN PROBLEM

The network design problem developed in this paper is defined over a directed graph $G = (N, A)$ where N denotes the set of nodes and A the set of arcs. $P \subset N$ is a set of possible locations for concentrators and $T \subset N$ is a set of terminal nodes. A root node $r \in N$ distributes flows to set T via a set $S \subseteq P$ of concentrators in a 2-level hierarchical network. The subnetwork in the first level is a Steiner network connecting the concentrators to the root node. The second level is itself a collection of disconnected Steiner subnetworks. Each Steiner subnetwork consists of a set of terminal nodes assigned to a concentrator. The capacity of a concentrator $p \in P$ can be set equal to the sum of the demands $d_t, t \in T$, for all terminal nodes reached from p. Therefore, the capacity of the root node can be set equal to the total demand. For each arc $(i, j) \in A$, there are nonnegative flows x_{ij} from i to j, variable costs c_{ij} and fixed costs f_{ij}. The fixed cost of locating a concentrator p is f_p. In the optimization problem, we need to find a subset of arcs and a subset $S \subseteq P$ of concentrators that minimize the sum of variable and fixed costs, (see Figure 1).

In this paper, we focus on two versions of the network design problem and we present a 3-phase algorithm. First, we locate the set $S \subseteq P$ of concentrators by using the uncapacitated facility location problem. Then, we treat the topological optimization problem which consists of finding a subgraph of the graph G defined at each level. The subgraph in Level-1 must be a tree consisting of the root node r and all the concentrators in S with a feasible flow of minimum cost. And, for each concentrator $p \in S$, the subgraph $G^p = (N^p\,, A^p)\,, N^p \subset N\,, A^p \subset A$, must also be a tree consisting of the origin node (concentrator p), and of all the terminal nodes reachable from p.

The algorithm to solve the uncapacitated location problem is based on Lagrangean relaxation and branch and bound methods. The topological optimization problems can also be solved by the same methods but, for practical purposes, we propose a simple heuristic.

The outline of the paper is as follows. We begin by presenting the structure of a 3-phase algorithm in section 3. Then, sections 4, 5 and 6 describe the approaches suggested for each phase of the algorithm. Finally, the computational results are analyzed and we present our conclusions.

3. ALGORITHM

In this section, we describe an algorithm for the proposed network design problem. Considering its complexity and flexibility, we adopt a divide-and-conquer strategy, solving smaller subproblems. The subproblems are well-known mathematical programming models, which makes this 3-phase approach easily implementable.

Let $G = (N, A)$ be a directed graph, $P \subset N$ be a set of possible locations for concentrators, $T \subset N$ be a set of terminal nodes and the root node $r \in N$. The 3-phase algorithm is of the following form:

Phase 1. Solve the uncapacitated location problem with the set $P \cup \{r\}$ of potential concentrator sites. Each arc $(i, j) \in A$ has a cost c_{ij}. The fixed cost to locate the concentrator $p \in P$ is f_p and $f_r = 0$. This algorithm generates a subset $S \subseteq P$ of concentrators and a subset T^p of terminal nodes assigned to the concentrator p, for all $p \in S$. We suppose that the designer has been able to fix a limit k on the number of concentrators, or $|S| \leq k$.

Phase 2. For each concentrator, $p \in S$ and T^p defined in Phase 1, look for a topology on the local network that minimizes the variable and fixed costs. The concentrator $p \in S$ is the origin node while the terminal nodes in T^p are demand nodes.

Phase 3. Solve the Level-1 topological network design problem where r is the origin node and the concentrators are demand nodes. The problem is similar to the Phase 2 model.

In the next section we emphasize the steps of the structure above. We present the formulations and algorithms.

4. CONCENTRATOR LOCATION - PHASE 1

The concentrator location problem is well-known in the literature, but the proposed algorithms generally do not take into account the Steiner nodes in the network wich is assumed in this paper. Other aspects to be analyzed are the capacities of the nodes and arcs. However, the Steiner nodes in the network can be implicitly and approximately considered if the minimal paths between pairs of non-Steiner nodes are calculated. Considering the new local network technologies available, the capacity of a concentrator can be as large as the total demand of the terminal nodes assigned to it. Then, given the network defined above, the concentrator location problem can be solved by the uncapacitated location problem (Bilde and Krarup, 1977; Erlenkotter, 1978; Christofides and Beasley, 1983; Beasley, 1985; Galvão and Raggi, 1989). In this case, we write the following mathematical formulation:

$$\min \sum_{i \in P \cup \{r\}} \sum_{j \in T} c_{ij} x_{ij} + \sum_{i \in P \cup \{r\}} f_i x_{ii} \tag{1}$$

subject to

$$\sum_{i \in P \cup \{r\}} x_{ij} = 1 \ , \forall j \in T \tag{2}$$

$$\sum_{i \in P \cup \{r\}} x_{ii} \leq k \tag{3}$$

$$x_{ij} \leq x_{ii} \ , \forall i \in P \cup \{r\} \ , \forall j \in T \tag{4}$$

$$x_{ij} \in \{0, 1\} \ , \forall i \in P \cup \{r\} \ , \forall j \in T \tag{5}$$

where x_{ii} indicates whether or not the node i is a concentrator. The variable x_{ij} assumes the value 1 if the demand of node j is met from i. c_{ij} is the cost (it may be the distance) to link i and j and f_i is the cost of locating the concentrator in node i. The constant k is the largest number of concentrators that can be installed.

The uncapacitated location problem is useful in this context and it enables one to solve large scale systems. However, its restricted formulation requires inflexible structures of costs and capacities. We need to adapt it, observing that only terminal nodes are demand nodes, and a concentrator can be connected to a demand node directly or by a Steiner node set. In order to overcome these fundamental difficulties, we can assume that the root node is directly connected to all nodes ($P \cup T$), and each node in P is directly connected to all nodes in T (see Figure 2). This assumption is possible, invoking a shortest path procedure where the costs are skillfully calculated.

The fixed cost f_i is a function of the shortest path between the candidate location node i and the root node:

$$f_i = \alpha_{ri} d_{ri} + g_i$$

where

f_i is the fixed cost to locate a facility in i,

α_{ri} is a constant for each arc (r, i),

d_{ri} is the shortest path between node i and the root node r, and

g_i is the hardware cost in the node i.

The constant α_{ri} enables one to represent the cost to link a concentrator at node i to the root node r. If the link exists, then $\alpha_{ri} = 0$. Otherwise,

$$\alpha_{ri} = cv_{ri} \bar{d} + cf_{ri}$$

where cv_{ri} is the variable cost per flow unit and per distance unit to link the root node to the concentrator at node i, $\bar{d}$ is the mean demand for each concentrator and cf_{ri} is the fixed cost per distance unit in the arc (r, i).

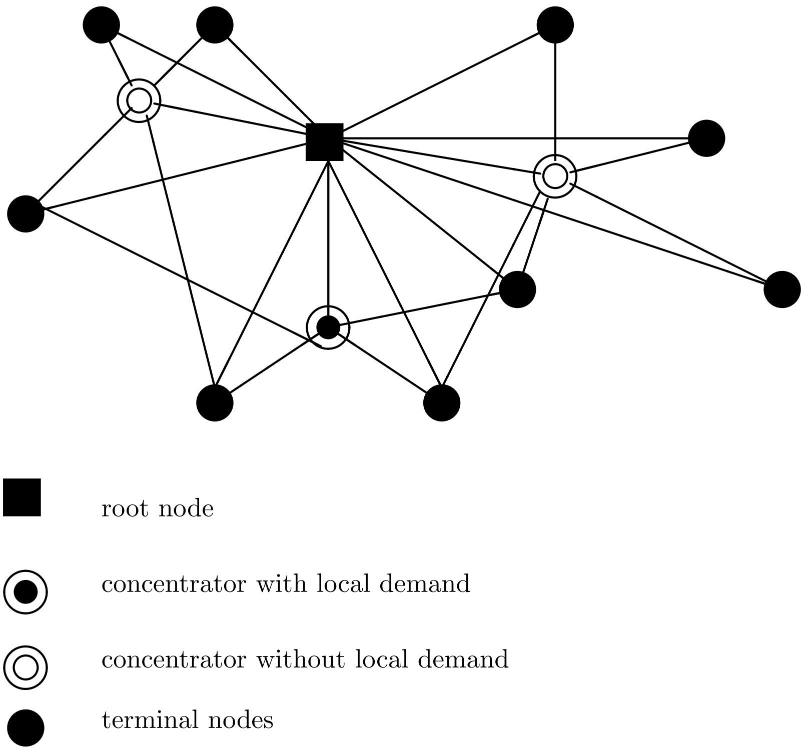

Figure 2. The uncapacitated location problem

On the other hand, the variable cost c_{ij} is a function of the shortest path between a candidate node i and a terminal node j:

$$c_{ij} = \beta_{ij} d_{ij}$$

where β_{ij} is a constant for each arc (i, j) and d_{ij} is the shortest path between nodes i and j.

For each Steiner node, the demand capacity is zero and c_{ij} is consequentely zero. For an existing arc, the constant β_{ij} can be zero, otherwise

$$\beta_{ij} = cv_{ij}\bar{d}_j + cf_{ij}$$

where

cv_{ij} is the variable cost per flow unit and per distance unit to connect concentrator i to terminal node j;

$\bar{d}_j$ is the demand in node j, and

cf_{ij} is the fixed cost per distance unit in arc (i, j).

The uncapacitated location problem is solved by the three phase algorithm proposed in Galvão and Raggi (1989).

Algorithm
  apply a primal-dual algorithm
  if solution is non-optimal then
      apply Lagrangean relaxation algorithm
      if solution is non-optimal then
          apply branch-and-bound algorithm
end algorithm.

The primal-dual phase is composed by a greedy heuristic (Galvão *et al.*, 1986), a dual ascent procedure to solve the dual problem (Erlenkotter, 1978) and a node substitution heuristic (Galvão *et al.*, 1986).

The second phase consists of an algorithm based on Lagrangean relaxation of the demand constraints (2) and uses the bounds of the first phase. The dual problem is solved by the subgradient method where, to determine the step size, the scalar λ_k is a continuous function of the iteration number.

Finally, if the dual gap persists, a branch-and-bound procedure is applied.

The primal-dual algorithm was developed to solve the general formulation of the uncapacitated location problem presented above and the computational results have shown a good performance of the algorithm. It was compared (Galvão and Raggi, 1989) with special purpose algorithms for the simple plant location (Erlenkotter, 1978), and for the k-median problem (Boffey and Karkazis, 1984; Christofides and Beasley, 1983; Beasley, 1985; Mateus and Carvalho, 1992), and it has also shown the same efficiency.

5. THE LOCAL NETWORK - PHASE 2

In this phase, the concentrators have been located and the set of terminal nodes assigned to each concentrator is known. Now, the goal is to establish the local network topology and dimensioning. The origin is a concentrator node with capacity equal to the sum of demands for all terminal nodes linked.

0.1 Local Network Models

Consider the subgraph $G^p = (N^p, A^p)$, $p \in S$, $N^p \subset N$, $A^p \subset A$, for the concentrator p, and the set T^p of terminal nodes. Each node $t \in T^p$ has a known demand d_t. Each arc $(i, j) \in A^p$ has a fixed cost f_{ij} and a variable cost c_{ij}. The local network optimization consists of finding a minimal subgraph of G^p, $G^{p'} = (N^{p'}, A^{p'})$, $N^{p'} \subset N^p$, $A^{p'} \subset A^p$, such that $G^{p'}$ is a tree consisting of p and all nodes of T^p and the installed capacity in each arc satisfies the demand constraints (see Figure 3).

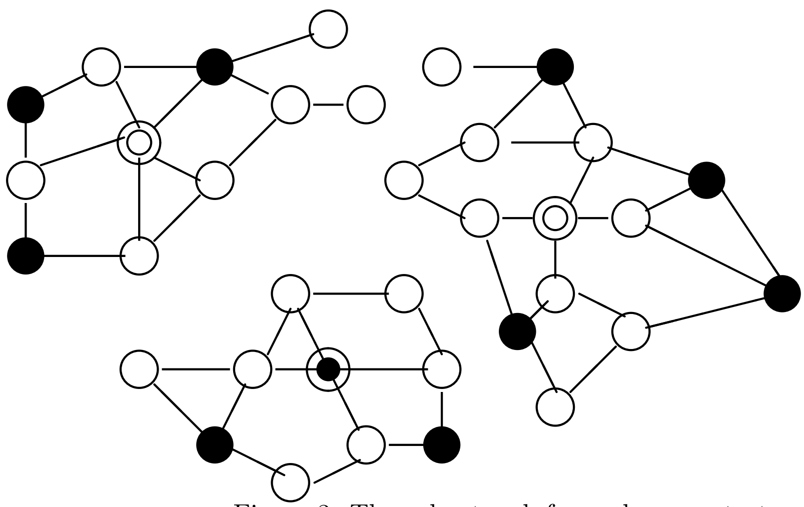

Figure 3. The subnetwork for each concentrator

The local network optimization problem is formulated as a mixed integer problem (M1):

$$\min \sum_{(i,j)\in A^p} c_{ij}x_{ij} + f_{ij}y_{ij} \qquad (6)$$

subject to

$$
\begin{aligned}
-\sum_{(p,k)\in A^p} x_{pk} &= -\sum_{t\in T^p} d_t && (7)\\
\sum_{(i,j)\in A^p} x_{ij} - \sum_{(j,k)\in A^p} x_{jk} &= 0\,,\ \forall j \in \{N^p - T^p - p\} && (8)\\
\sum_{(i,t)\in A^p} x_{it} - \sum_{(t,k)\in A^p} x_{tk} &= d_t\,,\ \forall t \in T^p && (9)\\
x_{ij} &\le M\,y_{ij}\,,\ \forall (i,j)\in A^p && (10)\\
x_{ij} &\ge 0\,,\ \forall (i,j)\in A^p && (11)\\
y_{ij} &\in \{0,1\}\,,\ \forall (i,j)\in A^p && (12)
\end{aligned}
$$

where x_{ij} is the flow through arc (i,j) and y_{ij} is a variable assuming a value one if arc (i,j) is chosen, zero otherwise.

The objective function (6) minimizes the variable and fixed costs. Constraint (7) ensures that the root node capacity is equal to the sum of demands in all terminal nodes. Constraints (8) are the usual network flow conservation equalities at each intermediate node, and constraints (9) impose the demand requirements. Constraints (10) express the fact that the flow through an arc must be zero if this arc is not included in the design, where M is a big number.

Although we present a subgraph $G^{p'}$ for each concentrator in Figure 3 and for the first level subnetwork in Figure 5, the computation of each subnetwork is defined over the original graph G, Figure 1.

A second formulation is obtained by reducing the graph G^p to $G' = (N', A')$ such that $N' = T^p \cup \{p\}$. In this case, we define a new special network, a complete graph, computing the shortest paths between all pairs of nodes in N'. It is easy to see that the new network corresponds to a situation where all Steiner nodes have been removed. The reduction is similar to the reduction process in Phase 1. Therefore, all the shortest paths are known and they can be useful again in this phase. With this assumption, the problem can be rewritten as (M2):

$$
\min \sum_{(i,j)\in A'} c_{ij}x_{ij} + f_{ij}y_{ij} \qquad (13)
$$

subject to

$$
\begin{aligned}
-\sum_{(p,t)\in A'} x_{pt} &= -\sum_{t\in T^p} d_t && (14)\\
\sum_{(i,t)\in A'} x_{it} - \sum_{(t,k)\in A'} x_{tk} &= d_t\,,\ \forall t \in T^p && (15)\\
x_{ij} &\le M\,y_{ij}\,,\ \forall (i,j)\in A' && (16)\\
\sum_{i\in(N'-t)} y_{it} &= 1\,,\ \forall t \in T^p && (17)\\
x_{ij} &\ge 0\,,\ \forall (i,j)\in A' && (18)\\
y_{ij} &\in \{0,1\}\,,\ \forall (i,j)\in A' && (19)
\end{aligned}
$$

where, for notational purpose, x also represents the new flows.

The second formulation includes a large number of optimum network design problems and it have been studied in the literature by Gavish (1982), Hochbaum and Segev (1989) and Minoux (1989). In practice, the use of shortest path algorithms has been shown to be a good strategy for the Steiner networks, which can confirm the effectiveness of this approach.

Both formulations are depicted in figure 4.

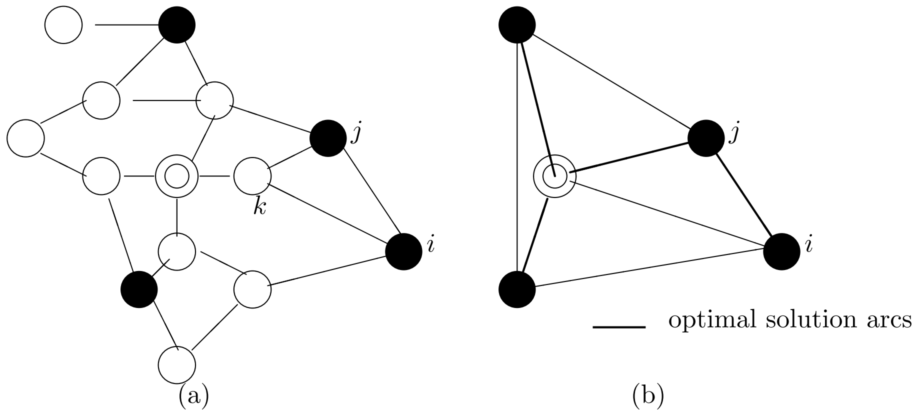

Figure 4. The original and approximate networks

0.2 Local Network Algorithm

In this section we propose a simple heuristic to solve Model (M1), or (M2):

Algorithm
reduce the original network to an approximate network
solve the approximate network problem
return to the original network
end algorithm.

Reduce the original network to an approximate network

This step consists of reducing the Model (M1), with the original network, to Model (M2) that it is associated with an approximate network. The reduction is obtained by the shortest path procedure as described before, (see Figure 4).

Solve the approximate network problem

To solve the approximate network problem, or Model (M2), we apply the following heuristic (Hochbaum and Segev, 1989):

Algorithm
apply a greedy heuristic
apply arc substitution heuristic
end algorithm.

- Greedy heuristic.

  A first feasible solution is obtained by a greedy heuristic that it is similar to Prim's procedure to solve the minimum spanning tree. This feasible solution is a tree that starts at the concentrator node p (or, at the root node r in the Level-1 subnetwork). At each iteration the arc (i, t), from the terminal node i to the terminal node t, with the minimal incremental cost $(c_{it}d_t + f_{it})$, is added to the tree. The process continues until a set of $(n-1)$ arcs has been selected.

- Arc substitution heuristic.

  The substitution heuristic tries to improve the greedy feasible solution by replacing an arc of the tree with an arc not in the tree. Let (i, t) the single input arc on node t. The procedure select an arc (j, t) with maximal positive saving of replacing (i, t) by (j, t). Saving is defined as the difference between the cost (variable plus fixed costs) of the tree with arc (i, t) and the cost of the new tree replacing (i, t) by (j, t). The process terminates if the maximal saving is not positive or if an imposed upper bound on the number of iterations is achieved.

The greedy/substitution heuristic is justified since we are working on the approximate network, eliminating Steiner nodes.

Return to original network

The step consists in identifying all the arcs in the shortest path between each terminal node and the concentrator (or root node). This search is not simple and we seek ways of reducing its complexity. Figure 4(b) shows the approximation for the original subnetwork of Figure 4(a) and the solution obtained. To return to the original network, node i must not be connected to node j directly, as was suggested by the solution of Figure 4(b). It is sufficient that node i be connected to node k, (see Figure 4(a)).

We always try to identify the arcs in the shortest path between the terminal node i to the origin (Figure 4(b)), passing through j. Although, if node k that belongs to the shortest path between j and the origin is reached, then the connection has been realized.

The algorithm defines a depth for each node. The origin node has a depth equal to zero. All the nodes directly connected to it have a depth of one and so on. The depth of a node is equal to the depth of the node that it is connected in the approximate solution increased by one. Hence, the nodes are connected to the origin node in increasing order of the depths. In addition, if a node i is connected to a node j in the approximate solution, then i is considered connected in the original network if a node k in the path between j and the origin node is reached.

6. THE FIRST LEVEL NETWORK

Phase 3 is similar to Phase 2. Now, the subgraph is $G^r = (N^r, A^r)$, where r is the root node with supply capacity equal $\sum_{t\in T} d_t$, $N^r \subset N$ and $A^r \subset A$. Each concentrator $p \in S$ is a demand node with capacity $\sum_{t\in T^p} d_t$ plus the local demand. Each arc $(i, j) \in A^r$ has a fixed cost f_{ij} and a variable cost c_{ij}.The cost of the subnetwork formed in Phase 2 around each concentrator is added to the objective function (see Figure 5).

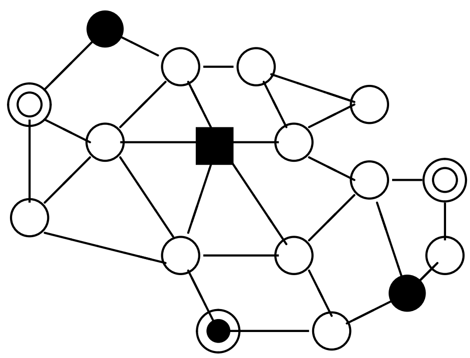

Figure 5. The first level network

The models and algorithms for this phase are the same of Phase 2.

Figure 6. Test problem - Network 3

Case	Number concent.	cf_{ij}	cv_{ij}	cf_{ri}	cv_{ri}	g_i	Level-2 network		Level-1 network	
							Upper bound	Time	Upper bound	Time
1	0	1	10	-	-	-	35790	0.2	-	-
2	0	1	20	-	-	-	69640	0.2	-	-
3	0	1	100	-	-	-	340440	0.2	-	-
4	8	1	10	1	20	-	18400	0.2	35340	0.1
5	8	1	10	1	100	-	35790	0.2	0	0.0
6	8	1	20	1	10	-	7560	0.1	31830	0.2
7	8	1	100	1	10	-	0	0.0	35790	0.2
8	8	1	20	1	10	500	7560	0.1	31830	0.2
9	8	1	20	1	10	5000	23205	0.2	24450	0.1

(a) Parameters, Phases 2 and 3 results

Case	Time 1	Upper bound	Lower bound	Time 2	Concentrators
1	1.6	37325	37325	3.3	1
2	1.6	71085	71085	3.2	1
3	1.6	341885	341885	3.2	1
4	1.6	33510	33510	3.4	34,22,36
5	1.6	37253	37253	3.3	1
6	1.6	29285	29285	2.6	34,38,22,43,35,36
7	1.6	37235	37235	2.2	34,38,22,43,35,36,25,39
8	1.6	32285	32285	2.6	34,38,22,43,35,36
9	1.6	50514	50007	4.8	34,38,22

(b) Concentrator location - Phase 1

Table 1: Network 1 results

7. COMPUTATIONAL RESULTS

We implemented the proposed heuristic in FORTRAN on a IBM 4341 computer. The test problems are three medium size brazilian cities (Figure 6 present Itajubá city network), where the number of nodes, arcs and terminal nodes are respectively (43,68,8), (74,87,46) and (189,297,50). The distances vary respectively from 40 to 340, from 1 to 432 and from 1 to 500. To study the convergence behavior of the algorithm, we construct a wide range of test cases with different cost structures. We analyze particular cases where the optimal locations are known a priori.

Table 1 contains the results for the first network, where Time 1 contains CPU seconds to define the complete graph and the shortest path between each pair of nodes. Time 2 contains CPU seconds spent by the uncapacitated location problem. A comparison of time shows that Phase 1 requires more computation time to solve the uncapacitated location problem than Phases 2 and 3.

For the first three cases, the number of concentrators is zero. The Level-1 network is degenerate and the root node is also the concentrator for the only Level-2 network. We reduce the problem to one hierarchy, connecting the terminal nodes directly in the root node. We increase the variable costs from 10 to 20 and 100 in Cases 2 and 3. The right side of part (a) in Table 1 reports bounds and times for Phases 2 and 3, and part (b) of Table 1 reports bounds and times for Phase 1.

The fourth case fixes a limit $k = 8$ on the number of concentrators. The parameters cf_{ij} and cf_{ri} are fixed equal to 1 in practice. The variable cost per flow unit and per distance unit (cv) is greater for the Level-1 network. With these parameters, the optimal solution consists in locating only one concentrator in the root node, since the Level-1 network is more expensive. This solution costs 35,790 (as for Case 1). However, this is not the solution obtained by our procedure that locates 3 concentrators.

The fifth case is similar to Case 4, but we increase the variable costs in Level-1 network. An optimal solution is obtained in this case. Our heuristic locates only one concentrator in the root node.

Case	Number concent.	cf_{ij}	cv_{ij}	cf_{ri}	cv_{ri}	g_i	Level-2 network		Level-1 network	
							Upper bound	Time	Upper bound	Time
1	0	1	10	-	-	-	823709	5.3	-	-
2	0	1	20	-	-	-	1637579	5.1	-	-
3	0	1	10	1	20	-	170443	3.3	1459743	0.3
4	46	1	20	1	10	-	172166	2.8	774637	0.6
5	46	1	20	1	10	1000	172166	2.9	774637	0.6
6	46	1	20	1	10	5000	172166	2.8	774637	0.6

(a) Parameters, Phases 2 and 3 results

Case	Time 1	Upper bound	Lower bound	Time 2	Concentrators
1	8.4	895257	895257	22.7	1
2	8.4	1709127	1709127	22.5	1
3	8.2	284165	284165	26.3	33,4,72,10
4	8.4	300889	300889	13.3	33,1,72,45,58,19,21,4,10
5	8.3	308889	308420	35.8	4,10,19,21,33,45,58,72
6	8.3	340880	338063	36.0	4,10,19,21,33,45,58,72

(b) Concentrator location - Phase 1

Table 2: Network 2 results

Cases 6 and 7 are also similar to Cases 4 and 5. Moreover, the variable costs for the Level-2 networks are greater than Level-1, as suggested by practical problems. The optimal solution consists in locating the maximal number of concentrators. In Case 7, the heuristic locates one concentrator for each terminal node. But, in Case 6, two Level-2 networks are generated to meet the demand in the terminal nodes 25 and 39. The value of this solution is (7,560 + 31,830), greater than 35,790.

Cases 8 and 9 include the fixed costs of locating concentrators. For Case 8, the following solutions are feasible:

1. the root node is the only concentrator (Case 2), the cost is 69,640;
2. six concentrators are located (Case 6), the cost is 7,560 + (6 $\times$ 500) = 42,390;
3. eight concentrators are located (Case 7), the cost is 35,790 + (8 $\times$ 500) = 39,790.

The heuristic solution does not differ appreciably from the best solution.

The following solutions are feasible for Case 9:

1. the root node is the only concentrator (Case 2), the cost is 69,640;
2. six concentrators are located (Case 6), the cost is 7,560 + 31,830 + (6 $\times$ 5,000) = 69,390;
3. eight concentrators (Case 7) with cost 35,790 + (8 $\times$ 5,000) = 75,790.

The heuristic selects three concentrators with cost 23,205 + 24,450 + (3 $\times$ 5,000) = 62,655, which is the best solution.

Table 2 summarizes the results for Network 2. The objective is to perform similar cases as in Network 1. We expect to locate the minimal number of concentrators in Case 3 and the maximal number in Case 4. In the last two cases, 5 and 6, we consider the fixed costs.

The fifth case presents the following feasible solutions:

1. the root node is the only concentrator, the cost is 1,637,579;
2. four concentrators are located, with cost 170,433 + 1,459,743 + (4 $\times$ 1,000) = 1,634,186;
3. nine concentrators, 172,166 + 774,637 + (9 $\times$ 1,000) = 955,803;
4. forty six concentrators, 823,709 + (46 $\times$ 1,000) = 869,709;

Case	Number concent.	cf_{ij}	cv_{ij}	cf_{ri}	cv_{ri}	g_i	Level-2 network		Level-1 network	
							Upper bound	Time	Upper bound	Time
1	0	1	10	-	-	-	262507	24.4	-	-
2	0	1	20	-	-	-	513877	24.1	-	-
3	0	1	10	1	20	-	109310	20.3	339819	4.2
4	50	1	20	1	10	-	105230	15.3	228394	10.2
5	50	1	20	1	10	1000	124020	17.6	224324	7.8
6	50	1	20	1	10	5000	160300	20.3	218594	5.1

(a) Parameters, Phases 2 and 3 results

Case	Time 1	Upper bound	Lower bound	Time 2	Concentrators
1	140.3	276507	276507	247.6	1
2	140.6	527877	527877	247.4	1
3	140.1	179576	178995	554.2	2,95,54,148,11,142,34,30,114,1
4	140.0	201998	201788	623.9	1,3,4,11,14,26,28,30,31,34,39,54, 75,92,96,98, 117,121,142,146,148,171,189
5	140.3	221190	212092	606.0	26,54,92,14,148,114,142,39,171, 31,75,146,99,11,28,3,86
6	139.7	278332	218541	601.3	2,54,95,13,148,114,142,39,171,31,75

(b) Concentrator location - Phase 1

Table 3: Network 3 results

5. the heuristic located eight concentrators with cost 172,166 + 774,637 + (8 × 1,000) = 954,803, which is a good solution.

For the sixth case, we can propose to locate:

1. one concentrator in the root node, the cost is 1,637,579;
2. four, the cost is 170,443 + 1,459,743 + (4 × 5,000) = 1,650,186;
3. nine, 172,166 + 774,637 + (9 × 5,000) = 991,803;
4. eight, 172,166 + 774,637 + (8 × 5,000) = 986,803;
5. forty six, 823,709 + (46 × 5,000) = 1,053,709.

The algorithm selects eight concentrators, which is the best of the solutions proposed.

Table 3 contains the results for Network 3. The basic parameters (part (a) of the table) are similar to Network 2.

For Case 3, the best solution is to locate only one concentrator with cost 262,507. The solution obtained costs 449,129. The solution is very expensive since the Level-1 network has been considered. On the other hand, a concentrator must be located at each terminal node in Case 4, with cost 262,507. The algorithm selects 23 concentrators with cost 333,624.

For Cases 5 and 6, we repeat the same tests of Network 2. For Case 5, the costs to locate 1, 10, 23 or 50 concentrators are 513,877; 459,129; 356,624 and 308,507 respectively. The algorithm locates 17 concentrators with cost 365,344.

For Case 6, the location of 1, 10, 23 or 50 concentrators costs 513,877; 499,129; 448,624 and 512,507 respectively. The heuristic proposes to locate 11 concentrators with cost 433,894, which is the best of the solutions proposed.

The results show that good solutions can be obtained with the proposed heuristic.

8. CONCLUSIONS

We treated the 2-level hierarchical network design problem and discussed a 3-phase algorithm. In the first phase, we applied the uncapacitated location problem to locate the concentrators. In the second and third phases, we presented two versions of the local network problem. The former is based on the Steiner tree problem and latter, based on a minimal spanning tree, is solved efficiently by the proposed heuristic.

Although the solutions obtained are acceptable from a practical point of view, other versions must be implemented for comparison. The computational results suggest a fast solution time, increasing with the number of nodes in the network. Moreover, the modularity of the system modeled offers a natural framework for decomposition methods.

As part of our practical objectives we are integrating network planning systems in a GIS - *Geographic Information System.* Graphical interfaces and database systems have improved the user interaction and provided new facilities for the network design process. They permit to treat the growth rate of data and information in the urban network.

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